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14. ABSTRACT
In this report we summarize research carried out during the period December 1, 2008 – November 30, 2011 under the sponsorship of the Air Force Office of Scientific Research under grant FA9550-08-1-0417. The results of our investigations were reported in 30 publications. They are listed on pages 4 – 6. Summaries of these publications are given on pages 7 – 13. Awards received during the grant period are noted on page 14. Scientists who have participated in this research are listed on page 15.

15. SUBJECT TERMS
Inverse problems, phase problems, degree of cross-polarization, statistical similarity, Gouy phase, Hanbury Brown-Twiss effect, polarization, phase anomaly, propagation of spectra

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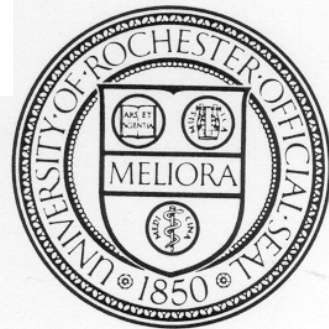
EFFECTS OF COHERENCE AND POLARIZATION IN RADIATION AND IN SCATTERING PROCESSES

Grant No: FA9550-08-1-0417

Dec 1, 2008 – Nov 30, 2011

Final Report

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LIST OF CONTENTS

I.   INTRODUCTION

In this report we summarize research carried out during the period December 1, 2008 – November 30, 2011 under the sponsorship of the Air Force Office of Scientific Research under grant FA9550-08-1-0417.

The results of our investigations were reported in 30 publications.   They are listed on pages 4 – 6. Summaries of these publications are given on pages 7 – 13. Awards received during the grant period are noted on page 14. Scientists who have participated in this research are listed on page 15.

II.   STATUS OF EFFORTS

During the period covered by this report we have fulfilled the research objectives listed in our Proposal. In particular, we obtained solution to the inverse problem in the theory of stochastic electromagnetic beams, elucidated how to determine correlation functions of stochastic media from scattering experiments and derived solution to an old and rather important problem in the theory of structure determination of crystals from diffraction experiments; namely we showed how the phase of the diffracted beams may be determined. We also introduced the concept of statistical similarity and showed that it provides a unifying viewpoint of the theories of coherence and of polarization of light.

We also found some new results relating to the so-called phase anomaly, which concerns the unusual behavior of the phase of light near focus.

We obtained some new results in the theory of correlations between intensity fluctuations of light at two space-time points (the so-called Hanbury Brown-Twiss effect). We also found equations which govern the propagation of spectra of non-stationary ensemble of pulses. We also obtained some new results relating to quantum properties of coherence and polarization of optical beams.

III. LIST OF PUBLICATIONS RESULTING FROM RESEARCH SUPPORTED BY GRANT FA9550-08-1-0417 DURING THE PERIOD DECEMBER 1, 2008 – NOVEMBER 30, 2011

1. D. Kuebel, M. Lahiri, and E. Wolf, "An Inverse Problem in the Theory of Stochastic Electromagnetic Beams", *Opt. Commun.* **282**, 141-142 (2008).

2. M. Lahiri and E. Wolf, "Cross-spectral Density Matrices of Polarized Light Beams", *Opt. Letts.* **34**, 551-553 (2009).

3. M. Lahiri, E. Wolf, D. G. Fischer and T. Shirai, "Determination of Correlation Functions of Scattering Potentials of Stochastic Media from Scattering Experiments", *Physical Rev. Letts.* **102**, 123901 (1-4), (2009).

4. E. Wolf, Reply to Comment on "Can a Light Beam be Considered to be the Sum of a Completely Polarized and a Completely Unpolarized Beam?" by Jani Tervo and Jari Turunen, *Opt. Letts.* **34**, 1002 (2009).

5. M. Lahiri and E. Wolf, "Spatial Coherence Properties of Monochromatic Electromagnetic Beams and of Laser Modes", *Phys. Letts. A*, **373**, 3694-3696, (2009).

6. E. Wolf, "Solution of the Phase Problem in the Theory of Structure Determination of Crystals from X-ray Diffraction Experiments", *Phys. Rev. Lett.* **103**, 075501 (1-3), (2009).

7. T. D. Visser, D. Kuebel, M. Lahiri, T. Shirai, and E. Wolf, "Unpolarized Light Beams with Different Coherence Properties", *Journal of Modern Optics* **56**, 1369-1374 (2009).

8. D. Kuebel, "Properties of the Degree of Cross-polarization in the Space-time Domain", *Opt. Commun.* **282**, 3397-3401 (2009).

9. M. Salem, and G. Agrawal, "Coupling of Stochastic Electromagnetic Beams into Optical Fibers", *Opt. Lett.* **34**, 2829-2831, (2009).

10. M. Lahiri, "Polarization Properties of Stochastic Light Beams in the Space-time and Space-frequency Domains", *Opt. Lett.* **34**, 2936-2938, (2009).

11. Lahiri and E. Wolf, "Beam Condition for Scattering on Random Media", *JOSA A*, **26**, 2043-2048 (2009).

12. E. Wolf, "Statistical Similarity as a Unifying Concept of the Theories of Coherence and Polarization of Light", *Opt. Commun.* **283**, 4427-4429, (2010).

13. E. Wolf, "Determination of Phases of Diffracted X-ray Beams in Investigations of Structure of Crystals", *Phys. Lett. A*, **374**, 491-495 (2010).

14.　T. D. Visser and E. Wolf, "The Origin of the Gouy Phase Anomaly and its Generalization to Astigmatic Wavefields", *Opt. Commun.*, **283**, 3371-3375 (2010).

15.　M. Lahiri and E. Wolf, "Does a Light Beam of Very Narrow Bandwidth Behave as a Monochromatic Beam?", *Phys. Lett. A,* **374**, 997-1000 (2010).

16.　T. van Dijk, D. G. Fischer, T. D. Visser, and E. Wolf, "The Effects of Spatial Coherence on the Angular Distribution of Radiant Intensity Generated by Scattering on a Sphere", *Phys. Rev. Lett.*, **104**, 173902 (1-4 pgs.) (2010).

17.　M. Lahiri and E. Wolf, "Relationship Between Complete Coherence Theory in the Space-Time and in the Space-Frequency Domains", *Phys. Rev. Lett.*, **105**, 063901-4 (2010).

18.　A. Al-Qasimi, M. Lahiri, D. Kuebel, D. F. V. James, and E. Wolf, "The Influence of the Degree of Cross-polarization on the Hanbury Brown-Twiss Effect", *Opt. Exp.*, **18**, 17124-9 (2010).

19.　M. Lahiri and E. Wolf, "Quantum Analysis of Polarization Properties of Optical Beams", *Phys. Rev. A.*, **82**, 043805-5 (2010).

20.　M. Salem and G. P. Agrawal, "Effects of Coherence and Polarization on the Coupling of Stochastic Electromagnetic Beams into Optical Fibers", *J. Opt. Soc. Am. A*, **26**, 2452-2458 (2009).

21.　M. Salem and J. Rolland, Heterodyne Efficiency of a Detection System for Partially Coherence Beams", *J. Opt. Soc. Am. A*, **27**, 1111-1119 (2010).

22.　E. Wolf, "History and Solution of the Phase Problem in the Theory of Structure Determination of Crystals from X-ray Diffraction Measurements", *Advances in Imaging and Electron Physics,* P. Hawkes, ed., Elsevier, Amsterdam, The Netherlands, **165**, 283-325 (2011).

23.　M. Lahiri and E. Wolf, "Quantum Theory of Optical Coherence of Nonstationary Light in the Space-frequency Domain", *Phys. Rev. A.* **82**, 04387-1-6 (2010).

24.　E. Wolf, "Solution of the Phase Problem of X-ray Crystallography", *Optics and Photonics News*, p. 30, Dec. 2010.

25.　E. Wolf, "What Kind of Phases Does One Measure in Usual Interference Experiments?", *Opt. Commun.*, **284**, 4235-4236, (2011).

26.　X. Pang, T. Visser, E. Wolf, "Phase Anomaly and Phase Singularities of the Field in the Focal Region of High-Numerical Aperture Systems", *Optics Commun.*, **284**, 5517-5522 (2011).

27.   M. Lahiri and E. Wolf, "Implications of Complete Coherence in the Space-frequency Domain", *Opt. Letts.*, **36**, 2423-2425 (2011).

28.   E. Wolf, "Interference Law for Coherent Stochastic Electromagnetic Beams", *Opt. Commun.*, **284**, 5240-5241 (2011).

29.   E. Wolf, "Equations that Govern the Propagation of Spectra of Non-stationary Ensembles of Pulses", *Opt. Commun.*, **285**, 941-942 (2011).

30.   M. Lahiri, "Wave-particle Duality and Polarization Properties of Light in Single-photon Interference Experiments", *Phys. Rev. A*, **83**, 045803 (2011).

IV.   SUMMARIES OF PUBLICATIONS RESULTING FROM RESEARCH SUPPORTED BY GRANT FA9550-08-1-0417 DURING THE PERIOD DECEMBER 1, 2008 – NOVEMBER 30, 2011

1. **D. Kuebel, M. Lahiri, and E. Wolf, "An Inverse Problem in the Theory of Stochastic Electromagnetic Beams", *Opt. Commun.* 282, 141-142 (2008).**
   The cross-spectral density matrix of an electromagnetic beam has been playing increasingly important role in studies of changes of spectra, of coherence and of polarization as the beam propagates. In this paper we derive solution to an inverse problem, which makes it possible to determine the cross-spectral density matrix of the beam in the source plane $z = 0$, from the knowledge of the matrix in any cross-section $z = z_0 > 0$ in the half-space into which the beam propagates. We apply the result to the theory of the so-called Stokes beams, which were introduced not long ago.

2. **M. Lahiri and E. Wolf, "Cross-spectral Density Matrices of Polarized Light Beams", *Opt. Letts.* 34, 551-553 (2009).**
   We show that there is no unique form of the cross-spectral density matrix of completely polarized light beams. We present three kinds of such matrices, each of which represents a beam that is completely polarized at every point. Some of the beams do not imitate monochromatic beams, in contrast to the usual assumption made in polarization optics.

3. **M. Lahiri, E. Wolf, D. G. Fischer and T. Shirai, "Determination of Correlation Functions of Scattering Potentials of Stochastic Media from Scattering Experiments", *Physical Rev. Letts.* 102, 123901 (1-4), (2009).**
   The classic ''Ewald-sphere construction'' for determining the structure of crystalline objects from x-ray and neutron diffraction experiments is generalized to determine the correlation functions of scattering potentials of stationary random media from scattering experiments.

4. **E. Wolf, Reply to Comment on "Can a light beam be considered to be the sum of a completely polarized and a completely unpolarized beam?" by Jani Tervo and Jari Turunen, *Opt. Letts*. 34, 1002 (2009).**
   The authors of the Comment [Opt. Lett. 34, 1001 (2009)] do not mention that no claim was made in the original letter that the cross-spectral density matrix of polarized light must be of a factorized form.

5. **M. Lahiri and E. Wolf, "Spatial coherence properties of monochromatic electromagnetic beams and of laser modes", *Phys. Letts. A*, 373, 3694-3696, (2009).**
   It is shown that, contrary to common belief, monochromatic light beams are, in general, not spatially completely coherent, i.e., they will, in general, not produce fringes of unit visibility in a Young's double pinhole interference experiment. We cite experiments with laser modes which confirm this result.

6. **E. Wolf, "Solution of the phase problem in the theory of structure determination of crystals from X-ray diffraction experiments", *Phys. Rev. Lett.* 103, 075501 (1-3), (2009).**

We present a solution to a long-standing basic problem encountered in the theory of structure determination of crystalline media from x-ray diffraction experiments; namely, the problem of determining phases of the diffracted beams.

7. **T. D. Visser, D. Kuebel, M. Lahiri, T. Shirai, and E. Wolf, "Unpolarized Light Beams with Different Coherence Properties", *Journal of Modern Optics* 56, 1369-1374 (2009).**
We investigate the coherence properties of unpolarized beams. Such beams form a much richer class than has been previously realized. We illustrate our results by examples.

8. **D. Kuebel, "Properties of the degree of cross-polarization in the space-time domain", *Opt. Commun.* 282, 3397-3401 (2009).**
A definition of the degree of cross-polarization in the space-time domain is introduced which is based on considerations of the correlations of intensity fluctuations of an electromagnetic beam at two points. Some general properties of this quantity are discussed. For the special cases of fully coherent and fully polarized light it is shown that the degree of cross-polarization exhibits a relationship between the states of polarization at the two points.

9. **M. Salem, and G. Agrawal, "Coupling of stochastic electromagnetic beams into optical fibers", *Opt. Lett.* 34, 2829-2831, (2009).**
We derive a general analytic expression for the coupling efficiency when a partially coherent, partially polarized beam is coupled into a multimode optical fiber. We adopt the Gaussian-Schell model for incident electromagnetic beams and use our general result to discuss the effects of the partial coherence and partial polarization on the coupling efficiency of an optical beam focused onto a step-index, single-mode fiber with a lens. Our results should be useful for any application requiring coupling of partially coherent beams into optical fibers.

10. **M. Lahiri, "Polarization properties of stochastic light beams in the space-time and space-frequency domains", *Opt. Lett.* 34, 2936-2938, (2009).**
Although the theories of polarization in the space-time and space-frequency domains are somewhat analogous, they have been developed independently, and there is no obvious connection between them. We investigate how they are related.

11. **M. Lahiri and E. Wolf, "Beam Condition for Scattering on Random Media", *JOSA A*, 26, 2043-2048 (2009).**
Properties of random media are frequently investigated by studying their interactions with stochastic electromagnetic fields. However, a stochastic beam does not necessarily retain its beamlike form on scattering, and the theory of stochastic electromagnetic fields that are not beamlike is rather complicated. In this paper necessary and sufficient conditions derived for a beam to retain its beamlike form after it is scattered on a stochastic medium. We illustrate the result by an example.

12. **E. Wolf, "Statistical Similarity as a Unifying Concept of the Theories of Coherence and Polarization of Light",** *Opt. Commun.*, **283, 4427-4429, (2010).**
It is shown that the concept of statistical similarity of light vibrations introduced in recent years provides a new insight into the physical meaning of coherence and of polarization of light and reveals a close analogy between the two phenomena.

13. **E. Wolf, "Determination of Phases of Diffracted X-ray Beams in Investigations of Structure of Crystals",** *Phys. Lett. A*, **374, 491-495 (2010).**
The phase problem of the theory of structure determination of crystals is reformulated within the framework of coherence theory; and it is shown that the reformation makes it possible to determine phases of diffracted beams from interference experiments.

14. **T. D. Visser and E. Wolf, "The Origin of the Gouy Phase Anomaly and its Generalization to Astigmatic Wavefields",** *Opt. Commun.*, **283, 3371-3375 (2010).**
One of the most poorly understood subjects in physical optics is the origin of the Gouy phase (sometimes called "the phase anomaly near focus"). This is evident from the large number of publications on the subject, many of which attribute it to quite different causes. In this paper we show that the Gouy phase anomaly can be clearly understood from elementary properties of normal congruences of light rays and from the relationship between geometrical optics and physical optics. We also show that the Gouy phase anomaly may be regarded as a degenerate case of a rapid $\pi/2$ phase change that is found to occur at each focal line of an astigmatic pencil of rays. The intensity distribution in the region of the phase changes is also presented. Furthermore, symmetry relations for both the phase anomaly and the intensity distribution are derived.

15. **M. Lahiri and E. Wolf, "Does a Light Beam of Very Narrow Bandwidth Behave as a Monochromatic Beam?",** *Phys. Lett. A,* **374, 997-1000 (2010).**
We show by analyzing coherence and polarization properties of narrow-band stochastic electromagnetic beams that a light beam of a very narrow bandwidth does not necessarily behave as a monochromatic beam with regards to its coherence and polarization properties. Our analysis also provides a new insight into the relation between the theories of coherence and polarization in the space–time domain and in the space-frequency domain.

16. **T. van Dijk, D. G. Fischer, T. D. Visser, and E. Wolf, "The Effects of Spatial Coherence on the Angular Distribution of Radiant Intensity Generated by Scattering on a Sphere",** *Phys. Rev. Lett.*, **104, 173902 (1-4 pgs.) (2010).**
In the analysis of light scattering on a sphere it is implicitly assumed that the incident field is spatially fully coherent. However, under usual circumstances the field is partially coherent. We generalize the partial waves expansion method to this situation and examine the influence of the degree of coherence of the incident field on the radiant intensity of the scattered field in the far zone. We show that when the coherence length of the incident field is comparable to, or is smaller than, the radius of the sphere, the angular distribution of the radiant intensity

depends strongly on the degree of coherence. The results have implications, for example, for scattering in the atmosphere and colloidal suspensions.

17. **M. Lahiri and E. Wolf, "Relationship Between Complete Coherence in the Space-Time and in the Space-Frequency Domains", *Phys. Rev. Lett.*, 105, 063901-4 (2010).**
We present some new results relating to properties of completely coherent optical fields. Our analysis elucidates the relationship between the theories of such fields in the space-time and in the space-frequency domains. We also show that the concept of cross-spectral purity, introduced by L. Mandel many years ago, plays an important role in clarifying this relationship.

18. **A. Al-Qasimi, M. Lahiri, D. Kuebel, D. F. V. James, and E. Wolf, "The Influence of the Degree of Cross-polarization on the Hanbury Brown-Twiss Effect", *Opt. Exp.*, 18, 17124-9 (2010).**
We show, by an example, that the knowledge of the degree of coherence and of the degree of polarization of a light beam incident on two photo-detectors is not adequate to predict correlations in the fluctuations of the currents generated in the detectors (the Hanbury Brown-Twiss effect). The knowledge of the so-called degree of cross-polarization, introduced not long ago, is also needed.

19. **M. Lahiri and E. Wolf, "Quantum Analysis of Polarization Properties of Optical Beams", *Phys. Rev. A.*, 82, 043805-5 (2010).**
We present a quantum theory of polarization of optical beams and discuss some properties of beams of any state of polarization. The analysis is based on quantum mechanical interpretation of a canonical experiment that is used to elucidate polarization properties of stochastic fields in classical optics. Our work shows how to apply some notions and techniques, commonly used in the classical theory, for fields that cannot be treated classically.

20. **M. Salem and G. P. Agrawal, "Effects of Coherence and Polarization on the Coupling of Stochastic Electromagnetic Beams into Optical Fibers", *J. Opt. Soc. Am. A.*, 26, 2452-2458 (2009).**
We study the problem of coupling an electromagnetic beam of any state of coherence and polarization into a multimode optical fiber. Using the well-known concept of the cross-spectral density matrix, we derive a general expression for the coupling efficiency of a stochastic electromagnetic beam into a multimode fiber in terms of the cross-spectral density matrix of the incident beam and another matrix representing field distributions of
fiber modes. We apply this result to a specific case in which the incident beam belongs to a broad class of so-called electromagnetic Gaussian Schell-model beams and obtain a simple analytical expression for the coupling efficiency in the case of single-mode fibers. We use this expression to study how coupling efficiency depends on the coherence and polarization properties of the incident beam.

21. **M. Salem and J. Rolland, Heterodyne Efficiency of a Detection System for Partially Coherence Beams", *J. Opt. Soc. Am. A*, 27, 1111-1119 (2010).**
We consider the heterodyne efficiency as a measure of quality for a coherent detection system. The heterodyne efficiency reflects the matching between the received beam and the local oscillator beam on the detector surface, and one can use this property for the alignment of the system. In this paper we derive a general expression for the heterodyne efficiency of a detection system for beams at any state of coherence, assuming that the propagation directions for the two signals (the received signal and the locally generated one) are slightly different. We derive an analytical expression for the heterodyne efficiency when mixing coherently two partially coherent Gaussian Schell-model beams on a photodetector surface. Numerical examples are given for the variation in the heterodyne efficiency with the misalignment angle, the detector radius, and the parameters of the overlapping beams. We show that partially coherent beams, although they suffer more than coherent beams from a decrease in the heterodyne efficiency, are less affected than coherent beams by the misalignment of the detection system.

22. **E. Wolf, "History and Solution of the Phase Problem in the Theory of Structure Determination of Crystals from X-ray Diffraction Measurements", *Advances in Imaging and Electron Physics,* P. Hawkes, ed., Elsevier, Amsterdam, The Netherlands, 165, 283-325 (2011).**
The subject reviewed in this article concerns a rather important old problem, first formulated about 100 years ago. It is of considerable interest in physics, chemistry, biology, and medicine. Its importance can be appreciated from the fact that about eleven Nobel Prizes were awarded, either for a partial solution of the problem or for the use of its approximate solution in specific applications. After a brief review of the history of research in this field, we present a solution of the problem obtained very recently (Wolf, 2009, 2010a).

23. **M. Lahiri and E. Wolf, "Quantum Theory of Optical Coherence of Nonstationary Light in the Space-frequency Domain", *Phys. Rev. A.* 82, 04387-1-6 (2010).**
Classical theories of coherence for statistically stationary, as well as nonstationary optical fields are frequently discussed both in the space-time and in the space-frequency domains. However, the quantum treatment of coherence theory is generally carried out in the space-time domain. In this paper,we present a quantum-mechanical theory of first-order coherence for statistically nonstationary light in the space-frequency domain.

24. **E. Wolf, "Solution of the Phase Problem of X-ray Crystallography", *Optics and Photonics News*, p. 30, Dec. 2010.**
A brief account is presented of recent research which has provided solution to an old problem of X-ray crystallography, the so-called phase problem. The knowledge of the phase of X-ray beams diffracted by crystalline solids together with measurements of the average intensity of the diffracted beams makes it possible to determine structure of the crystals. The importance of this problem is

evident from the fact that 11 Nobel Prizes have been awarded for partial solution of the problem and for the use of the technique to estimate structures of various crystalline solids. The most spectacular use of this technique was the determination of the structure of DNA molecules which carries information about heredity.

25.   **E. Wolf, "What Kind of Phases Does One Measure in Usual Interference Experiments?",** *Opt. Commun.*, **284, 4235-4236, (2011).**
It is pointed out that in usual optical interference experiments one does not measure phases of monochromatic light beams as is usually assumed. Rather that one measures a phase of an associated correlation function of a beam; and that, when the beam is spatially fully coherent, as is usually the case, the measured phase may be identified with the phase of an average wave function of the beam.

26.   **X. Pang, T. Visser, E. Wolf, "Phase Anomaly and Phase Singularities of the Field in the Focal Region of High-Numerical Aperture Systems",** *Opt. Commun.*, **284, 5517-5522 (2011).**
The phase behavior of the three Cartesian components of the electric field in the focal region of a high-numerical aperture focusing system is studied. The Gouy phase anomaly and the occurrence of phase singularities are examined in detail. It is found that the three field components exhibit different behaviors.

27.   **M. Lahiri and E. Wolf, "Implications of Complete Coherence in the Space-frequency Domain",** *Opt. Letts.*, **36, 2423-2425 (2011).**
It was shown not long ago that complete spatial coherence of light at a pair of points in the space–time domain may be interpreted as a manifestation of so-called "statistical similarity" between the fluctuating field at the two points. In this Letter, we consider complete spatial coherence at a pair of points in the space–frequency domain and derive a condition that the field at those points must obey. We illustrate the usefulness of the condition by an example.

28.   **E. Wolf, "Interference Law for Coherent Stochastic Electromagnetic Beams",** *Opt. Commun.*, **284, 5240-5241 (2011).**
Two-beam interference law for the superposition of stochastic, spatially coherent, electromagnetic beams is derived. Only a single phase is found to enter the interference law, in spite of the vector nature of the problem. The meaning of the phase is elucidated.

29.   **E. Wolf, "Equations that Govern the Propagation of Spectra of Non-stationary Ensembles of Pulses",** *Opt. Commun.*, **285, 941-942 (2011).**
Formulas are derived, which make it possible to study the changes produced in the spectrum of a non-stationary ensemble of pulses on propagation in free space or in a linear medium, whether deterministic or random.

30.   **M. Lahiri, "Wave-particle Duality and Polarization Properties of Light in Single-photon Interference Experiments",** *Phys. Rev. A*, **83, 045803 (2011).**

We consider superposition of two states of light polarized along mutually orthogonal directions. We show that partial polarization of the superposed light may be interpreted as a manifestation of the wave-particle duality.

VII. AWARDS RECEIVED BY THE PRINCIPAL INVESTIGATOR OF THIS PROPOSAL DURING THE GRANT PERIOD

Faculty Lifetime Achievement Award (The University of Rochester, Hajim School of Engineering and Applied Sciences, 2009).

G. G. Stokes Award [SPIE (Society of Photo-illumination Engineering) Awards Committee, 2010] in recognition of contributions in formulating the modern theories of coherence and polarization of optical fields.

VIII. SCIENTIFIC COLLABORATORS

In addition to Professor Emil Wolf, the Principal Investigator for this Proposal, the following scientists have taken part in the research:

AGRAWAL, G. P.   Professor, Institute of Optics, University of Rochester, Rochester, NY.

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